Received April 24, 1767.

XXVII. *An Inquiry into the probable Parallax, and Magnitude of the fixed Stars, from the Quantity of Light which they afford us, and the particular Circumstances of their Situation, by the Rev.* John Michell, *B. D. F. R. S.*

Read May 7, and 14, 1767.

THOUGH no man can at present doubt, that the want of a sensible parallax in the fixed stars, is owing to their immense distance, yet it may not perhaps be disagreeable to see, that this distance is farther confirmed by other circumstances; for let us suppose them to be, at a medium, equal in magnitude and natural brightness to the sun, to which they seem in all respects to be analagous. And, having laid this down as a foundation to build upon, let us inquire what would be the parallax of the sun, if he were to be removed so far from us, as to make the quantity of the light, which we should then receive from him, no more than equal to that of the fixed stars. In order to do this with accuracy, it would be proper to compare the quantity of light; which we at present receive from him, with that of the fixed stars, by some such methods, as are made use of by Monsieur Bouguer

Bouguer in his *Traité d'Optique* [a]; but as my preſent purpoſe does not require any ſuch exactneſs, I ſhall deduce it in a more groſs way from facts already well known. I ſhall aſſume Saturn then in oppoſition, excluſively of his ring (and when the earth and he are at their mean diſtances from the Sun) as equal or nearly equal in light to the moſt luminous fixed ſtar. Now the mean diſtance of Saturn from the Sun, being equal to about 2082 of the Sun's ſemidiameters, the denſity of the Sun's light, at Saturn, will conſequently be leſs than at his own ſurface, in the proportion of the ſquare of 2082 (or 4334724) to 1. If Saturn therefore was to reflect all the light, that falls upon him, he would be leſs luminous in the ſame proportion; but, beſides this difference in his brightneſs, his apparent diameter, in oppoſition, is at moſt but 105th part of that of the Sun, and conſequently the quantity of light, which we receive from him, muſt again be diminiſhed in the proportion of the ſquare of 105 (or 11025) to 1. If we multiply theſe two numbers together, we ſhall have the whole of the light of the Sun to that of Saturn (upon the ſuppoſition of his reflecting all the light, that falls upon him) as the ſquare of nearly 220000 (or 48,400,000,000) to 1; and removing the Sun to 220000 times his preſent diſtance, he would ſtill appear at leaſt as bright as Saturn, and his whole parallax upon the diameter of the earth's orbit would be leſs than two ſeconds. This muſt conſequently be aſſumed for the parallax of the brighteſt of the fixed ſtars, upon the ſuppoſition that their light does not exceed that of Saturn.

[a] This work was publiſhed at Paris in 1760.

By a like computation we ſhall find, that the diſtance, at which the Sun would afford us as much light, as we receive from Jupiter, is not leſs than 46000 times his preſent diſtance, and his whole parallax, in that caſe, upon the diameter of the earth's orbit, would not be more than nine ſeconds, the light of Jupiter and Saturn, as ſeen from the Earth, being in the ratio of about 22 to 1, when they are both in oppoſition, and ſuppoſing them to reflect equally in proportion to the whole of the light that falls upon them.

But if Jupiter and Saturn, inſtead of reflecting the whole of the light, that falls upon them, ſhould in fact reflect only a part of it, as for example, only a fourth or ſixth (and this may very poſſibly be the caſe), we muſt then increaſe the diſtances computed above, in the proportion of 2 or 2½ to 1, to make the Sun's light no more than equal to theirs; and his parallax would be leſs, in the ſame proportion, than thoſe already mentioned [a].

Upon the ſuppoſition then, that the fixed ſtars are of the ſame magnitude and brightneſs with the Sun, it is no wonder, that their parallax ſhould have hitherto eſcaped obſervation, ſince, if this is the caſe, it could hardly amount to two ſeconds, and probably

[a] The light, which we receive from the full Moon (according to Monſieur Bouguer's experiments in the work above-mentioned), is only a 300000th part of that which we receive from the Sun, whereas it ought to amount to no leſs than a 45000th part of it, according to the principles, which we have made uſe of in computing the quantity of light derived from Jupiter and Saturn; ſo that the Moon, as appears from theſe experiments, reflects no more than between a ſixth and a ſeventh part of the light that falls upon her.

not more than one in Sirius himſelf; though he had been placed in the pole [b] of the ecliptic, and in thoſe, that appear much leſs luminous, ſuch for example as γ draconis, which is only of the third magnitude, it could hardly be expected to be ſenſible with ſuch inſtruments as have hitherto been uſed in ſearch of it.

We have aſſumed the magnitude of the fixed ſtars, as well as their brightneſs, to be equal to thoſe of the ſun; it is however probable, that there may be a very great difference amongſt them in both theſe reſpects; and how much ſoever we may therefore be wide of the truth, in attempting to fix the diſtance of particular ſtars from this reaſoning, yet there is a very great probability, that their mean diſtances, ſettled by this method, will not be much out, ſome exceeding and ſome falling ſhort of it. And perhaps the conſideration, that a ſtar muſt be a thouſand times as great, *cæteris paribus*, to appear equally bright, if it is placed at ten times the diſtance, may ſerve to make it probable, that the limits of the errors, which we are likely to commit, in judging by ſuch a rule, are not ſo great as we might otherwiſe imagine them to be.

With regard to the difference there may be in the native brightneſs of different ſtars, though it is probably very conſiderable, yet I think we can hardly ſuppoſe, that it is equal to their difference in magnitude, at leaſt if we except thoſe, which are ſubject to certain changes, and which for that reaſon we may ſuppoſe to be luminous in ſome parts of their ſurfaces

[b] The latitude of Sirius being only 39° 33′, his parallax will be a little leſs than two thirds of the whole parallax.

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only. In other inſtances we may perhaps judge in ſome degree of the native brightneſs of different ſtars with reſpect to one another by their colour; thoſe, which afford the whiteſt light, being probably the moſt luminous [a].

[a] We have at preſent no means of judging of the comparative brightneſs of the Sun and of the fixed ſtars, in proportion to their reſpective ſizes, excepting from the compariſon of the Sun's brightneſs with that of our common fires; but the Sun's light exceeds the light of our brighteſt fires in ſo very great a proportion, (viz. of ſome thouſands to one) that we want ſome middle terms to be able to form any analogy, which might ſerve to carry us farther. We find however in general, that thoſe fires, which produce the whiteſt light, are much the brighteſt, and that the Sun, which produces a whiter light than any fires we commonly make, vaſtly exceeds them all in brightneſs; it is not therefore improbable, from this general analogy, that thoſe ſtars, which exceed the Sun in the whiteneſs of their light, may alſo exceed him in their native brightneſs; now this is the caſe with regard to many of them; and, on the contrary, there are ſome that are of a redder colour.

If however it ſhould hereafter be found, that any of the ſtars have others revolving about them (for no ſatellites ſhining by a borrowed light could poſſibly be viſible), we ſhould then have the means of diſcovering the proportion between the light of the Sun, and the light of thoſe ſtars, relatively to their reſpective quantities of matter; for in this caſe, the times of the revolutions, and the greateſt apparent elongations of thoſe ſtars, that revolved about the others as ſatellites, being known, the relation between the apparent diameters and the denſities of the central ſtars would be given, whatever was their diſtance from us: and the actual quantity of matter which they contained would be known, whenever their diſtance was known, being greater or leſs in the proportion of the cube of that diſtance. Hence, ſuppoſing them to be of the ſame denſity with the Sun, the proportion of the brightneſs of their ſurfaces, compared with that of the Sun, would be known from the compariſon of the whole of the light which we receive from them, with that which we receive from the Sun; but, if they ſhould happen to be either of greater or leſs denſity than the Sun, the whole of their light not being

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As far then as we can guess at the parallax of the fixed stars from the principles above laid down, we may reasonably expect, that it should be exceedingly small even in those of the first magnitude; yet, besides the probability, that some of them may be either less, or less luminous than the Sun, it is not so small as to leave us altogether without hopes, that we may some time or other be able to discover it in some of them; for I think it is not impracticable to construct instruments, capable of distinguishing even to the 20th part of a second, provided the air will admit of that degree of exactness; but such instruments must be upon a plan a good deal different from those hitherto made use of, as they would otherwise be not only vastly too expensive, but also much too great and unweildy to be of any use.

But whatever room there may be to hope, that we may some time or other be able to discover the parallax of a few amongst the fixed stars, yet at the same time it seems probable, that we shall never be able to discover any sensible magnitude in their apparent diameters, which in Sirius himself, if he is not of less native brightness than the Sun, must be considerably less, whatever be his parallax, than the

affected by these suppositions, their surfaces would indeed be more or less luminous, accordingly as they were, upon this account, less or greater; but the quantity of light, corresponding to the same quantity of matter, would still remain the same.

The apparent distances, at which satellites would revolve about any stars, would be equal to the semiannual parallaxes of those stars, seen from planets revolving about the Sun, in the same periodical times with themselves, supposing the parallaxes to be such, as they would be, if the stars were of the same size and density with the Sun.

hundreth, probably than the two hundreth part of a ſecond; ſo that it would ſcarcely be diſtinguiſhable with a teleſcope, upon the former ſuppoſition, that ſhould magnify ſix, or, upon the latter with one, that ſhould magnify twelve thouſand times.

Nor can we well expect to find their apparent diameters from any occultation by the moon, ſince a diameter of the hundredth part of a ſecond would be covered by the moon, if it entered directly, in leſs than the fiftieth part of a ſecond of time, and therefore a ſtar can hardly enter ſo obliquely, as to appear to vaniſh by degrees; no ſtar probably, which the moon can paſs over, ſubtending an angle half ſo great. A ſtar might however appear to vaniſh by degrees in an occultation by the planet Venus, eſpecially if the occultation was to happen only a little before or after either ſtation; but this is an event, which can occur ſo very ſeldom, that little is to be expected from it; and if Venus ſhould be ſurrounded with an atmoſphere, which is probably the caſe, it might very poſſibly then be of no ſervice at all. For the ſame reaſons alſo it is probable, that nothing can be determined from occultations by any of the reſt of the planets, which upon other accounts are ſtill leſs proper for the purpoſe than Venus.

There ſeems to be little chance therefore of diſcovering with certainty the real ſize of any of the fixed ſtars, and we muſt conſequently be contented to deduce it from their parallax (if that ſhould ever be found) and the quantity of light which they afford us, compared with that of the Sun. And in the mean time, till this parallax can be found, or ſomething elſe may ariſe to furniſh us with a more general analogy, we can only ſuppoſe them, at a medium, to be equal in ſize to the Sun, this being the beſt means, which we have at preſent of forming ſome probable conjecture concerning the extent of the viſible univerſe. That we may be the better enabled to do this, it ſeems to be an object worth the attention of Aſtronomers, to enquire into the exact quantity of light, which each ſtar affords us ſeparately, when compared with the Sun; that, inſtead of diſtributing them, as has hitherto been done, into a few ill defined claſſes, they may be ranked with preciſion both according to their reſpective brightneſs, and the exact degree of it.

A catalogue of the ſtars formed upon this plan, would anſwer ſeveral good purpoſes; for, beſides giving us a more juſt and certain notion of their general diſtances, it would perhaps help us, eſpecially if the parallax of a few amongſt them ſhould be diſcovered likewiſe, to trace ſome analogies in their ſituation, which might enable us to determine both their diſtances, and other circumſtances relating to them, with ſtill more probability; and it would be a ſtanding regiſter, by which future Aſtronomers might inform themſelves of many variations, of which we are now ignorant for want of an ancient regiſter of that kind.

From what has been ſaid above, I think it is very probable, that we ſhall not be a great way from the truth in eſtimating the whole parallax of Sirius at one ſecond, ſuppoſing him to be of the ſame ſize and native brightneſs with the Sun; this therefore I ſhall aſſume as a ſtandard, till ſome better experiments ſhall inform us more exactly of the quantity of his light.

light. Now, according to the beſt judgment I have been able to make from ſome groſs experiments, the quantity of light which we receive from Sirius does not exceed the light which we receive from the leaſt fixed ſtars of the ſixth magnitude in a greater proportion than that of 1000 to 1, nor in a leſs proportion than that of 400 to 1; and the ſmaller ſtars of the ſecond magnitude ſeem to be about a mean proportional between the two. Hence the whole parallax of the leaſt fixed ſtars of the ſixth magnitude, ſuppoſing them of the ſame ſize and native brightneſs with the Sun, ſhould be from about 2‴ to 3‴, and their diſtance from about eight to twelve million times that of the Sun: and the parallax of the ſmaller ſtars of the ſecond magnitude, upon the ſame ſuppoſition, ſhould be about 12‴, and their diſtance about two million times that of the Sun.

I have hitherto argued about the diſtances of the fixed ſtars, upon the ſuppoſition of their being of the ſame ſize and brightneſs with the Sun; and, if this was really ſo, thoſe which appear the brighteſt muſt be the neareſt to us. That this is in general the caſe, I ſuppoſe, will be very readily allowed; for, though it is true, that a much greater degree of real magnitude may compenſate for the greatneſs of diſtance, and there is no reaſon for aſſigning any one limit to them rather than another; yet, when it is as likely that the largeſt ſtars ſhould be in any one part of ſpace as in any other, the probability in favour of this hypotheſis is very great: the real motions alſo, which have been obſerved amongſt ſeveral of the brighteſt of the fixed ſtars, is another argument to the ſame purpoſe; and we ſhall find it ſtill farther confirmed

confirmed by very ſtrong arguments of analogy drawn from the circumſtances of the particular ſituation of the ſtars in the heavens.

It has always been uſual with Aſtronomers to diſpoſe the fixed ſtars into conſtellations: this has been done for the ſake of remembering and diſtinguiſhing them, and therefore it has in general been done merely arbitrarily, and with this view only; nature herſelf however ſeems to have diſtinguiſhed them into groups. What I mean is, that, from the apparent ſituation of the ſtars in the heavens, there is the higheſt probability, that, either by the original act of the Creator, or in conſequence of ſome general law (ſuch perhaps as gravity) they are collected together in great numbers in ſome parts of ſpace, whileſt in others there are either few or none.

The argument, I intend to make uſe of, in order to prove this, is of that kind, which infers either deſign, or ſome general law, from a general analogy, and the greatneſs of the odds againſt things having been in the preſent ſituation, if it was not owing to ſome ſuch cauſe.

Let us then examine what it is probable would have been the leaſt apparent diſtance of any two or more ſtars, any where in the whole heavens, upon the ſuppoſition that they had been ſcattered by mere chance, as it might happen. Now it is manifeſt, upon this ſuppoſition, that every ſtar being as likely to be in any one ſituation as another, the probability, that any one particular ſtar ſhould happen to be within a certain diſtance (as for example one degree) of any other given ſtar, would be repreſented (according to the common way of computing chances) by a fraction,

 whoſe

whoſe numerator would be to it's denominator, as a circle of one degree radius, to a circle, whoſe radius is the diameter of a great circle (this laſt quantity being equal to the whole ſurface of the ſphere) that is, by the fraction $\frac{\overline{60'}^{2}}{\overline{6875.5'}^{2}}$, or, reducing it to a decimal form, .000076154 (that is, about 1 in 13131) and the complement of this to unity, viz. .999923846, or the fraction $\frac{13130}{13131}$, will repreſent the probability that it would not be ſo. But, becauſe there is the ſame chance for any one ſtar to be within the diſtance of one degree from any given ſtar, as for every other, multiplying this fraction into itſelf as many times as ſhall be equivalent to the whole number of ſtars, of not leſs brightneſs than thoſe in queſtion, and putting n for this number, $\overline{.999923846}^{n}$, or the fraction $\overline{\frac{13130}{13131}}^{n}$ will repreſent the probability, that no one of the whole number of ſtars n would be within one degree from the propoſed given ſtar; and the complement of this quantity to unity will repreſent the probability, that there would be ſome one ſtar or more, out of the whole number n, within the diſtance of one degree from the given ſtar. And farther, becauſe the ſame event is equally likely to happen to any one ſtar as to any other, and therefore any one of the whole number of ſtars n might as well have been taken for the given ſtar as any other, we muſt again repeat the laſt found chance n times, and conſequently

consequently the number $\overline{.999923846n}|^{n}$, or the fraction $\overline{\frac{13130n}{13131n}}|^{n}$ will represent the probability, that no where, in the whole heavens, any two stars, amongst those in question, would be within the distance of one degree from each other; and the complement of this quantity to unity will represent the probability of the contrary.

By a like reasoning, if we would compute the probability, upon the same supposition, that no two stars should be, one within the given distance x, and the other within the given distance z of some one particular star, we must, first, find the probability, that no star, of the whole number of stars n, would be within the distance x from the given star, which will be represented, as before, by the fraction $\overline{\frac{\overline{6875.5'}^{2}-xx}{\overline{6875.5'}^{2}}}|^{n}$ and, secondly, we must find the probability, that no star, of the whole number of stars n, would be within the distance z from the given star, which, for the same reason, will be represented by the fraction $\overline{\frac{\overline{6875.5'}^{2}-zz}{\overline{6875.5'}^{2}}}|^{n}$; and the complements of these to unity will represent the respective probabilities of the contrary: but the probability that two events shall both happen, is the product of the respective probabilities of those two events multiplied together; if therefore we multiply the two last mentioned complements together, we shall have the probability, that some two stars would be, one within the distance x, and the other

other within the diſtance z from the given ſtar; and the complement of this to unity, will be the probability, that it would not be ſo: let us now ſuppoſe $\frac{c}{d}$ to repreſent this laſt quantity, and, becauſe the ſame event may as well happen in reſpect to any one ſtar, as any other, multiplying this quantity into itſelf n times, according to the number of the ſtars, we ſhall have $\overline{\frac{c}{d}}|^{n}$ repreſenting the probability, that no where, in the whole heavens, would be found any two ſtars, one within the diſtance x, and the other within the diſtance z from the ſame ſtar.

If now we compute, according to the principles above laid down, what the probability is, that no two ſtars, in the whole heavens, ſhould have been within ſo ſmall a diſtance from each other, as the two ſtars β Capricorni, to which I ſhall ſuppoſe about 230 ſtars only to be equal in brightneſs, we ſhall find it to be about 80 to 1.

For an example, where more than two ſtars are concerned, we may take the ſix brighteſt of the Pleiades, and, ſuppoſing the whole number of thoſe ſtars, which are equal in ſplendor to the fainteſt of theſe, to be about 1500, we ſhall find the odds to be near 500000 to 1, that no ſix ſtars, out of that number, ſcattered at random, in the whole heavens, would be within ſo ſmall a diſtance from each other, as the Pleiades are[a].

[a] The computations of theſe probabilities are as follow.

The diſtance between the two ſtars β Capricorni is ſomething leſs than $3'\frac{1}{3}$; according to the rule above laid down, therefore, if we ſuppoſe 230 ſtars equal to theſe in brightneſs, the proba-

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If, besides these examples that are obvious to the naked eye, we extend the same argument to the smaller

bility, that no two stars of that brightness will be found, any where in the whole heavens, within the distance of $3' \frac{1}{3}$ from each other, will be represented by the fraction $\overline{\frac{\overline{6875.5}^2 - \overline{3.33\ \&c.}^2}{\overline{6875.5}^2}}\Big|^{230 \times 230}$.

From twice the Log. of 6875.5 [7.6746086] then, subtract twice the Log. of 3.33 &c. [1.0457496] and the remainder 6.6288590 will be the Log. of the number of times, that $\overline{3.33\ \&c.}^2$ is contained in $\overline{6875.5}^2$ viz. 4254603 times, and consequently $\overline{\frac{4254602}{4254603}}\Big|^{230 \times 230}$ will be equivalent to the former fraction. From the Log. of 4254602, subtract the Log. of 4254603, and the remainder will be—.000000102, the proportional part for an unit in the number 4254603: this multiplied into 230 times 230, or 52900, gives —.0053958, the Log. of the whole quantity, which corresponds to the proportional part for an unit between 80 and 81; this quantity therefore is equivalent to the fraction $\frac{80}{81}$ nearly, the complement of which to unity is $\frac{1}{81}$.

In the Pleiades, the five stars Taygeta, Electra, Merope, Alcyone, and Atlas are respectively at the distances of 11, $19\frac{1}{2}$, $24\frac{1}{2}$, 27, and 49 minutes from the star Maia nearly. From 7.6746086, twice the Log. of 6875.5, then, as before, subtract 2.0827854, twice the Log. of 11 (the number of minutes between Taygeta and Maia) and the remainder 5.5918232 will be the Log. of the number of times, that $\overline{11}^2$ is contained in $\overline{6875.5}^2$ viz. 390682 times; and consequently a fraction, whose denominator is this number, and whose numerator is this number less by an unit, multiplied into itself 1500 times, will represent the probability, that no star out of 1500, scattered by chance in the whole heavens, would be within the distance of 11 minutes from the star Maia. From the Log. of 390681 therefore subtract the Log. of 390682, as in the former example, and the remainder will be—.00000111, the proportional part for an unit in the number 390682, which multiplied by 1500 will give us—.0016650 for the Log. of the probability sought. In like manner from

stars,

ſtars, as well thoſe that are collected together in cluſters, ſuch for example, as the Præſepe Cancri,

7.6746086 ſubtract 2 5800692, twice the Log. of 19½ (the number of minutes between Electra and Maia) and the remainder will be 5 0945394, the proportional part for an unit correſponding to the natural number of which will be—.00000349; and 1500 times this quantity, or—.0052350 will be the Log. of the quantity, repreſenting the probability, that no ſtar out of 1500 ſcattered by chance would be within the diſtance of 19½ minutes from Maia. If we follow the ſame rule for the three remaining ſtars Merope, Alcyone, and Atlas, we ſhall find the ſimilar Logs. correſponding to theſe to be—.0076650, — .0100395, and —.0330300 reſpectively. The natural numbers correſponding to theſe five Logs. taken in the ſame order, are .996173, .988018, .982506, .977148, and .926766, which ſeverally expreſs the reſpective probabilities, that no ſtars out of 1500 ſcattered by chance, would be within the ſame diſtances, at which the five ſtars above mentioned are found to be, from Maia. The complements of theſe quantities to unity .003827, .011982, .017494, .022852, and .073234, which ſeverally expreſs the reſpective probabilities, on the contrary, that ſuch ſtars would be found within the diſtances above ſpecified from the ſtar Maia, muſt all be multiplied together, to determine the probability, that theſe events ſhould all take place at the ſame time. The ſum of the Logs. of theſe numbers is—9 + .1279139 or—8.8720861, which is therefore the Log. of all theſe numbers multiplied together; and ſubtracting this number from 0, or, what amounts to the ſame thing, changing the ſign, we ſhall have the Log. of the number of times that this quantity is contained in unity, that is, about 744880000 times; a fraction, therefore, whoſe denominator is this number, and its numerator unity, will repreſent the probability in favour of all theſe events taking place together; and a fraction, whoſe denominator is the ſame number, and its numerator the ſame number leſs by an unit, will repreſent the probability of the contrary. But, as this event might as well have happened to any other ſtar as to Maia, we muſt multiply this laſt fraction into itſelf 1500 times, according to the ſuppoſed number of ſtars, to find the probability, that it ſhould not have happened any where in the whole heavens. Now the proportional part for an unit upon this number is—.0000000005825, which multiplied the

the nebula in the hilt of Perſeus's ſword, &c. as to thoſe ſtars, which appear double, treble, &c. when ſeen through teleſcopes, we ſhall find it ſtill infinitely more concluſive, both in the particular inſtances, and in the general analogy, ariſing from the frequency of them.

We may from hence, therefore, with the higheſt probability conclude (the odds againſt the contrary opinion being many million millions to one) that the ſtars are really collected together in cluſters in ſome places, where they form a kind of ſyſtems, whilſt in others there are either few or none of them, to whatever cauſe this may be owing, whether to their mutual gravitation, or to ſome other law or appointment of the Creator. And the natural concluſion from hence is, that it is highly probable in particular, and next to a certainty in general, that ſuch double ſtars, &c. as appear to conſiſt of two or more ſtars placed very near together, do really conſiſt of ſtars placed near together, and under the influence of ſome general law, whenever the pro-

by 1500 gives us—.000000087377, the proportional part for an unit in ſomewhat more than 496000.

But it muſt be obſerved, that this number is ſmaller than it ought to be upon two accounts; for, in the firſt place, this method of computation gives only the probability, that no five ſtars would be within the diſtance above ſpecified from a ſixth, if they occupied the largeſt ſpace, they poſſibly could do, under that limitation; and ſecondly, we have made no allowance upon account of the different magnitudes, which, if it had been attended to, would have given a ſomewhat greater reſult. Theſe conſiderations, however would have made the reaſoning a good deal more intricate; and we have no need to deſcend to minutiæ, a difference in the proportion of 10 to 1 not at all affecting the general concluſion.

 bability

bability is very great, that there would not have been any ſuch ſtars ſo near together, if all thoſe, that are not leſs bright than themſelves, had been ſcattered at random through the whole heavens.

After what has been ſaid, it will be natural to inquire, whether, if the ſtars are in general collected into ſyſtems, the Sun does not likewiſe make one of ſome ſyſtem; and which are thoſe, amongſt the fixed ſtars, that belong to the ſame ſyſtem with himſelf. Now, ſuppoſing the ſtars of one ſyſtem to be, in general and at a medium, of the ſame ſize and brightneſs with thoſe of another, the number of ſtars of any one apparent magnitude would bear the ſame proportion to the number of ſtars of any other apparent magnitude, as they would do, in caſe all the ſtars were ſcattered uniformly, and not in ſyſtems, provided the eye was not placed in or near to one of thoſe ſyſtems. And, in this caſe, the brightneſs decreaſing, as the ſquare of the diſtance inverſely, and the ſphere, in which they are included, increaſing, as the cube of the diſtance directly, the number of ſtars of any one degree of brightneſs and upwards, ſhould be, as the cube of the ſquare root of that brightneſs. Suppoſing then the fainteſt of the 2000 brighteſt ſtars to be leſs bright than the fainteſt of the firſt 70, in the proportion of about 30 to 1 (and I think the difference is not leſs than this) this number is ſmaller, than we might have expected, if the Sun was not one of a ſyſtem, in the proportion of 2000 to about 12000 or 1 to 6; but I ſhall lay the leſs ſtreſs upon this argument, for want of a more certain determination of the proportion of light, which we receive from the ſtars of different magnitudes.

If,

If, however, upon a more accurate examination, it ſhould be found, that the quantity of light above aſſigned is not far from the truth, or if the difference of light ſhould be greater than I have ſuppoſed it to be (in which caſe the argument will be ſtill ſtronger); this will add a conſiderable degree of weight to the other arguments drawn from analogy, in favour of the Sun's making one of a ſyſtem of ſtars.

If we would now inquire, which are probably thoſe ſtars, which compoſe part of the ſame ſyſtem with the Sun; though it will not be poſſible to point them out with certainty, yet there are ſome marks, by which we may, with great probability include ſome and exclude others, whilſt the reſt remain more doubtful. Thoſe ſtars which are found in cluſters and ſurrounded with many others at a ſmall diſtance from them, belong probably to other ſyſtems and not to ours. And thoſe ſtars, which are ſurrounded with nebulæ, are probably only very great ſtars, which, upon account of their ſuperior magnitude, are ſingly viſible, whilſt the others, which compoſe the remaining parts of the ſame ſyſtem, are ſo ſmall as to eſcape our ſight. And thoſe nebulæ, in which we can diſcover either none, or only a few ſtars, even with the aſſiſtance of the beſt teleſcopes, are probably ſyſtems, that are ſtill more diſtant than the reſt.

The Pleiades, as they appear to the naked eye, have been ſhewn above to be probably a ſyſtem by themſelves; and if we examine them ſtill farther by means of teleſcopes, we ſhall find, that they are ſurrounded with ſo large a number of ſmaller ſtars, as to increaſe the odds againſt the contrary opinion many millions to one. Now ſuppoſing the Pleiades to be in reality a ſyſtem of ſtars, the probability is at leaſt, I ſuppoſe, an hundred to one, that no one amongſt them, of thoſe viſible to the naked eye, belongs to the ſame ſyſtem with the Sun; but that theſe are only ſuch ſtars, as are greater than the reſt. The exact quantity of this probability depends upon the number of ſtars, viſible to the naked eye, belonging to this ſyſtem; the proportion, that the ſpace occupied by the Pleiades bears to the whole heavens; and laſtly, how far the ſituation of any one of the Pleiades falls in with the general analogy of the ſtars compoſing this ſyſtem, if any ſuch general analogy ſhould appear.

As the nebulæ, and ſmaller conſtellations, compoſed of a great number of ſtars, within a ſmall diſtance from one another, belong probably to other ſyſtems; ſo thoſe, which being placed at greater diſtances from each other compoſe the larger conſtellations, and ſuch as have few or no ſmaller ſtars near them, when examined with teleſcopes, belong probably to our own ſyſtem. Moſt of the ſtars of the firſt and ſecond magnitude have this criterion to diſtinguiſh them as belonging to the ſame ſyſtem with the Sun, beſides ſeveral other circumſtances, ſuch as their greater brightneſs; the proper motions [a], that have

[a] The apparent change of ſituation, that has been obſerved amongſt a few of the ſtars, is a ſtrong circumſtance in favour of thoſe ſtars being ſome of the neareſt to us. This apparent change of ſituation may be owing either to the real motion of the ſtars themſelves, or to that of the Sun, or partly to the one, and partly to the other. As far as it is owing to the latter (which it is by no means improbable may in ſome meaſure be the caſe) it may be conſidered as a kind of ſecular parallax, which, if the annual parallax of a few of the ſtars ſhould ſome time or other be diſcovered, and the quantity and direction of the Sun's motion

been obſerved amongſt ſome of them; their being more numerous, than we might naturally expect in proportion to the ſmaller ſtars, if they did not compoſe a part of the ſame ſyſtem with ourſelves, &c.

Beſides the brighter ſtars, it is probable there are many of thoſe of the ſmaller magnitudes alſo, that belong to the ſame ſyſtem with the Sun. Now, if this ſhould be the caſe, many of them being only fainter upon account of their leſs real magnitude, we may ſtand the ſame chance of finding a parallax amongſt ſome of theſe, as amongſt the brigheſt ones, provided we pitch upon ſuch as happen to belong to our own *ſyſtem*: to direct us with ſome probability to theſe, we have the circumſtances above-mentioned of their compoſing larger conſtellations, and their having few ſtars lying very near to them; and perhaps there may be ſtill a little more reaſon to ſuſpect, that thoſe ſtars form a part of the ſame ſyſtem with ourſelves, where, beſides theſe circumſtances, there have been obſerved changes of brightneſs, &c. amongſt ſeveral of them in the ſame neighbourhood, ſuch for inſtance as the changes, which have been obſerved amongſt ſeveral of the ſtars in the conſtellations of the Swan, Caſſiopeia, &c. We may, I think, alſo venture to add to theſe moſt of thoſe ſtars, that are of a redder hue than the reſt, as it is probable that they are a good deal bigger, in proportion to their brightneſs, than the whiter ſtars [ſee the laſt note

ſhould be diſcovered likewiſe, might ſerve to inform us of the diſtances of many of them, which it would be utterly impoſſible to find out by any other means.

but

but two]. Many of them alſo have been obſerved to have a proper motion of their own, which with ſeveral other concurrent circumſtances tends to make it highly probable, that they are ſome of the neareſt to us.

Having thus endeavoured to eſtabliſh the probability, that the Sun is one of a ſyſtem of ſtars, placed by themſelves in this part of the univerſe, I ſhall next inquire into ſome of the conſequences of this hypotheſis. Now if this is the caſe, and we ſuppoſe the whole number of ſtars, which belong to this ſyſtem only (excluding thoſe which belong to others), to amount to about 1000, we ſhall find it to be an even chance, that the parallax of the neareſt amongſt them does not exceed the parallax of one half that number in a greater proportion than that of 9 to 1, ſuppoſing the Earth to be placed in or near the centre of the whole ſyſtem; nor in a greater proportion than that of about 12 to 1, ſuppoſing it to be placed very near the edge of the ſyſtem; for ſuppoſing 1000 points to be placed within a ſphere of any given magnitude, and that they are equally indifferent to every part of that ſphere, if we aſſume any one of theſe points as a centre, we ſhall find, according to the known doctrine of chances, that there is an equal degree of probability whether any one of the reſt ſhall or ſhall not fall within a ſphere, deſcribed about the point aſſumed, of about ſeven ten thouſandths of the ſize of the whole ſphere; but the radius of ſuch a ſphere is about $\frac{8}{90}$ or a little leſs than $\frac{1}{11}$ of the radius of the whole ſphere, that is about $\frac{1}{9}$ of the radius of a ſphere of half the ſize of the greater one; and therefore a ſphere, of about nine times this radius, will

will include half the greater ſphere, if its centre be aſſumed near the centre of the greater ſphere, and a ſphere of twelve times this radius will include half the greater ſphere, if the point be aſſumed almoſt in the ſurface of it.

If we aſſume the ſtars belonging to our own ſyſtem to be about 1000; ſince they are in all reſpects ſimilar to the Sun (excepting perhaps ſuch amongſt them as are liable to frequent changes), and we have nothing to determine us to one magnitude rather than another, we may moſt reaſonably ſuppoſe the magnitude of the Sun to be a medium amongſt the whole number. Upon this ſuppoſition, he will probably rank only with the ſtars of the fourth magnitude; and his light therefore, if he was removed to the medium diſtance of the other ſtars (viz. a diſtance equal to the radius of the ſphere, that would include half the ſtars of our own ſyſtem) would hardly, I think, exceed a 200th part of the light of Sirius; and conſequently, if the parallax of Sirius would be about one ſecond, if he was of the ſame ſize and native brightneſs with the Sun, it will be an equal chance, that the parallax of one half of the ſtars, belonging to this ſyſtem, is not leſs than one ſecond divided by the ſquare root of 200, that is a little more than 4‴; and it will likewiſe be an equal chance, that the parallax of no one amongſt them exceeds between 9 and 12 times that quantity, or a little more than two thirds of a ſecond.

If, inſtead of 1000 ſtars, we ſuppoſe the whole number, belonging to this ſyſtem, to be only about 350, the Sun will then, if he is of a medium ſize amongſt them, rank probably with the ſtars of the

third

third magnitude, and his light, at a medium diſtance, upon this hypotheſis, would be, I apprehend, about a 50th part of that of Sirius. And therefore, according to the reaſoning above, we ſhould then find it an equal chance, that the parallax of one half of theſe 350 ſtars would not be leſs than about 8‴ $\frac{1}{2}$; and there would be the ſame chance, that the parallax of no one amongſt them would be more than between 50‴ and about 1″.

In the former ſuppoſition of 1000 ſtars; the apparent magnitude of the Sun, when removed to the medium diſtance; &c. it ſeems not improbable, that the biggeſt ſtar in the ſyſtem may perhaps exceed the Sun, in the proportion of about 1000 to 1; and in the latter ſuppoſition of 350 ſtars, &c. that it may perhaps exceed the Sun, in the proportion of about 120 to 1.

In whatever proportion the diameter of the Sun is greater or leſs than the medium we have taken for it in the ſuppoſitions above, in the ſame proportion will the parallaxes be increaſed or diminiſhed; and in the inverſe triplicate of that proportion muſt their magnitudes be diminiſhed or increaſed.

Let us now examine the circumſtances of the Pleiades; and, aſſuming the reſpective diſtances of the ſtars, compoſing that ſyſtem, from each other to be, at a medium, equal to thoſe of our own, let us ſee what will be the conſequences of this ſuppoſition. Now, if the Pleiades do not extend farther in the direction of a line drawn between the Earth and them, than in a direction at right angles to that line (which, from their compoſing a ſyſtem, we have a right to ſuppoſe they do not) we can hardly allow the mean

diſtance

diſtance of thoſe, that are next to each other, amongſt the ſix ſtars viſible to the naked eye, to be greater than what would ſubtend an angle, if ſeen directly from the earth, of about forty or fifty minutes. And conſequently the diſtance between them and the earth would be about 70 times that diſtance, and their apparent brightneſs, ſeen from thoſe that are next to each other, muſt be about 4900 times as great, as it appears to us; But η of the Pleiades, if I judge rightly, is not fainter than Sirius in a greater proportion than that of about 100 to 1; this ſtar therefore muſt appear brighter to the neareſt of thoſe ſix, which are viſible to the naked eye, than Sirius does to us, in the proportion of about 49 to 1. Let us now ſuppoſe all the ſtars belonging to the Pleiades, as well thoſe diſcoverable with teleſcopes, as thoſe which are viſible to the naked eye, to be contained within a ſphere, whoſe apparent diameter at the earth is two degrees; and conſequently the mean diſtance of them from a ſpectator placed ſomewhere amongſt them, as it might happen, would ſubtend an angle, when ſeen directly from the earth, of about a degree. Since therefore we have ſuppoſed the diſtances of the ſtars of our own ſyſtem to be, at a medium, equal to thoſe of the Pleiades, and conſequently their mean diſtances from the earth to ſubtend at the Pleiades an angle of one degree, we ſhall have the diſtance of the Pleiades about 57 times as great as the mean diſtance of the ſtars of our own ſyſtem, from the earth. Hence, if the biggeſt of the ſtars of our own ſyſtem ſhould be at this mean diſtance from us, and Sirius ſhould be the biggeſt, η of the Pleiades muſt exceed it in the proportion of about 200 to to 1; for removing Sirius to 57 times his present distance, his light would then be fainter than it is, in the proportion of 3249 to 1, that is, fainter than η of the Pleiades in the proportion of 32.49 to 1, supposing η of the Pleiades, as above, to afford us a hundredth part of the light of Sirius; but the magnitude of stars, supposing them equally luminous and their distance to be given, is as the cube of the square root of their brightness; and therefore η of the Pleiades, upon this supposition, must be bigger than Sirius in the proportion of the cube of the square root of 32.49 (that is 185) to 1.

But I must observe, that according to general (and, I believe, I may say universal) analogy in all those nebulæ, in which we discover stars bigger than the rest, these stars are placed towards the middle of their respective systems, and, if therefore the same thing obtains with regard to our own system, this will make η of the Pleiades still something greater.

If the distance of the Pleiades is greater than the mean distance of the stars of our own system, in the proportion of 57 to 1, it would be necessary, in order to make stars, of the same real magnitude amongst the Pleiades, equally visible to us with those of our own system, to take in a pencil of rays of a greater diameter, than the pupil of the eye, in the same proportion: this, after proper deductions for the loss of light, could not well be effected by an object lens of less than two feet aperture. Now Dr. Hooke tells us, in his Micrographia, that, with a telescope of twelve feet length, he discovered in the Pleiades 78 stars, and, with longer telescopes, many more; but if a teleſcope of twelve feet length, the aperture of whoſe object lens was probably leſs than two inches, increaſed the number of viſible ſtars in the Pleiades to 78, we may well ſuppoſe, that with an object lens of two feet diameter, they would amount to more than 1000. What this number would be muſt depend however upon the gradation of real magnitude amongſt the ſtars of that ſyſtem, to which there muſt neceſſarily be ſome limit, and it is not therefore improbable, that obſervations of the increaſe of the number of ſtars amongſt the Pleiades, &c. with teleſcopes of larger apertures, eſpecially if this was carried on to very large ſizes, might ſerve to inform us of many circumſtances, both with regard to this gradation, and perhaps ſome other things, that would enable us to judge with more probability concerning the diſtances, magnitudes, &c. of the ſtars of our own ſyſtem.

If we now imagine a ſpectator amongſt the Pleiades to take a view of this ſyſtem from thence, ſuppoſing the diſtance, as before, 57 times the mean diſtance of our own ſtars, we ſhould appear to him as a nebula, in which there would be no ſtar bright enough to be diſtinguiſhable by the naked eye; and with a teleſcope, the aperture of whoſe object lens was two inches, he would hardly, I ſuppoſe, be able to diſtinguiſh more than half a ſcore ſtars at the utmoſt.

Having hitherto ſuppoſed the diſtances of the ſtars of our own ſyſtem to be the ſame with thoſe of the Pleiades, and examined the appearances according to that hypotheſis, let us now, inſtead of their diſtances, ſuppoſe their magnitudes to be the ſame. This would make this ſyſtem, as ſeen from the Pleiades, to ſubtend an angle of about twelve degrees inſtead of two, and about half a ſcore of the biggeſt ſtars would be there viſible to the naked eye; but a teleſcope, whoſe object lens was of two inches aperture, would in that caſe, I apprehend, take in almoſt all the ſtars belonging to this ſyſtem of the fourth magnitude and upwards. Theſe appearances fall in leſs, I think, with the general analogy of what we ſee in the heavens, than the former ſuppoſition; but for want of more obſervations I cannot ſay this with any certainty: in the mean time however till we have ſomething farther to go upon, it may not perhaps be amiſs to take a kind of medium between the two, and ſuppoſe the Pleiades to be at about twenty times the mean diſtance of the ſtars belonging to our own ſyſtem, in which caſe η will exceed the biggeſt of theſe, in the proportion of about eight or ten to one; or it will exceed the Sun, according to our former ſuppoſitions of his being of a medium ſize amongſt 1000 or 350 ſtars, in one caſe in the proportion of about eight or ten thouſand, and in the other, about a thouſand or twelve hundred to one; its parallax in the former caſe being about $36''''$ and in the latter about $1\frac{1}{4}'''$.

I ſhall conclude this inquiry with one obſervation more, concerning the appearance of the ſtars of our own ſyſtem, as ſeen from great diſtances. Whatever then the real diſtance and magnitude of theſe ſtars may be, provided we have not been greatly out in aſſigning the proportion of their light in reſpect to that of the Sun and one another, if they were to be ſeen from a diſtance, at which the whole ſyſtem would not ſubtend an angle of more than ſix or eight minutes, it would appear only as a nebula, no ſingle ſtar being viſible with perhaps any teleſcope, that has ever yet been made; for at this diſtance, the diſtance between the earth and the biggeſt ſtar of this ſyſtem not ſubtending an angle of more than about three minutes (that is, about a twelve hundredth part of the radius) the ſtars of this ſyſtem muſt appear leſs luminous than they do to ourſelves, in the proportion of the ſquare of 1200 (or 1440000) to 1. And ſuppoſing the light of Sirius to exceed that of the leaſt viſible ſtar in the proportion of about 1200 to 1, the brighteſt ſtar therefore would ſtill require to have its light increaſed in the proportion of 1200 to 1, before it would begin to be diſtinguiſhable. To do this would require a teleſcope, that ſhould take in a pencil of rays of a larger diameter than the pupil of the eye, in the proportion of 35 to 1, that is, a pencil of about a foot diameter, excluſive of deductions; for the pupil of the eye is not leſs than a third of an inch in diameter, in a clear ſtar-light night, when there is no Moon; but to obtain ſuch a pencil, we muſt not make uſe of a refracting teleſcope (with two lenſes only) of leſs than 15 inches, nor a reflector of leſs than nearly two feet aperture. This may ſerve to ſhew us, that thoſe nebulæ, in which we cannot diſtinguiſh any ſtars, may yet reaſonably be ſuppoſed to conſiſt of ſtars, though too far diſtant to be ſingly viſible; ſince this would be the caſe with our own ſyſtem, ſeen from as great a diſtance as we may well ſuppoſe thoſe nebulæ to be from us, if we judge of it from the magnitude of the viſible area, which they occupy in the heavens.

Of the twinkling of the fixed ſtars.

Having never yet ſeen any ſolution of the twinkling of the fixed ſtars, with which I could reſt ſatisfied,

fied [a], I ſhall offer the following, which may not perhaps be found an inadequate cauſe of that appearance; at leaſt it has undoubtedly ſome ſhare in producing it, eſpecially in the ſmaller ſtars.

It is not, I think, unreaſonable to ſuppoſe, that a ſingle particle of light is ſufficient to make a ſenſible impreſſion upon the organs of ſight. Upon this ſuppoſition, a very few particles of light, arriving at the eye in a ſecond of time, will be ſufficient to make an object viſible, perhaps not more than three or four; for though the impreſſion may be conſidered as momentary, yet the perception, occaſioned by it, is of a much longer duration: this ſufficiently appears from the well-known experiment of a lighted body whirled round in a circle, which needs not make many revolutions in a ſecond, to appear as one continued ring of fire. Hence then it is not improbable, that the number of the particles of light, which enter the eye in a ſecond of time even from Sirius himſelf, may not exceed three or four thouſand; and from ſtars of the ſecond magnitude, they may therefore probably not much exceed an hundred. Now the apparent increaſe and diminution of the light, which we obſerve in the twinkling of the ſtars, ſeems to be repeated at not very unequal intervals, perhaps about four or five times in a ſecond: why may we not then ſuppoſe, that the in-

[a] Some aſtronomers have lately adopted, as a ſolution of this appearance, the extreme minuteneſs of the apparent diameters of the fixed ſtars, which, they ſuppoſe, muſt, in conſequence of this, be intercepted by every little mote, that floats in the air; but, that an object ſhould be able to intercept a ſtar from us, it muſt be large enough to exceed the apparent diameter of the ſtar by the diameter of the pupil of the eye; ſo that, if the ſtar was a mathematical point, it muſt ſtill be equal in ſize to the pupil of the eye.

equalities,

equalities, which will naturally ariſe from the chance of the rays coming ſometimes a little denſer and ſometimes a little rarer, in ſo ſmall a number of them as muſt fall upon the eye in the fourth or fifth part of a ſecond, may be ſufficient to account for this appearance? An addition of two or three particles of light, or perhaps of a ſingle one upon twenty, eſpecially if there ſhould be an equal deficiency out of the next twenty, would I ſuppoſe be very ſenſible: this ſeems at leaſt probable from the very great difference in the appearance of ſtars, whoſe light is much leſs different than, I imagine, people are in general aware of; the light of the middle-moſt ſtar in the tail of the great Bear does not, I think, exceed the light of the very ſmall ſtar next to it, in a greater proportion than that of about ſixteen or twenty to one; and Monſieur Bouger tells us, in his Traité d'Optique before-mentioned, that he finds a difference in the light of objects of one part in ſixty-ſix ſufficiently diſtinguiſhable.

It will perhaps be objected, that the rays coming from Sirius are too numerous to admit of a ſufficient inequality, ariſing from the common effect of chance, ſo frequently as would be neceſſary to produce this effect, whatever might happen in reſpect to the ſmaller ſtars; but till we know what inequality is neceſſary to produce this effect, we can only gueſs at it either one way or the other; there is however another circumſtance, that ſeems to concur in the twinkling of the ſtars, beſides their brightneſs, and this is a change of colour. Now the red and blue rays being very much fewer, I apprehend, than thoſe of the intermediate colours, and therefore much more liable to inequality from the common effect of chance, may help very much to account for this phænomenon, a ſmall exceſs or defect in either of these making a very sensible difference in the colour.

It will now naturally be asked, why the frequency of the changes of brightness should not be often much greater, as well as sometimes less, than that above-mentioned, and why the interval of the fourth or fifth, or some such part, should be pitched upon, rather than the fortieth or fiftieth part of a second, or than a whole second, &c. for, according to the length or shortness of the time assumed, the changes, that will naturally occur, from the effect of chance, will be smaller or greater in proportion to each other. The answer to this question will, I think, tend to render the above solution more probable, as well as to throw a good deal of light upon the whole subject. The lengths of the times then between the changes of brightness, if I am not mistaken, depend upon the duration of the perception before mentioned, occasioned by the impression of the light upon the eye, than which they seem to be neither much longer nor shorter. Whatever inequalities fall within a much shorter time than the continuance of this perception, will necessarily be blended together, and have no effect, but as they compose a part of the whole mass; but those inequalities, which fall in such a manner as that they may be assigned to intervals nearly equal to, or something greater than the continuance of this perception, will be so divided by the imagination, which will naturally follow, and pick them out as they arise.

END OF PART I.